Bihar Mathematical Society

TSTM Examination (Junior) Solution 2019

Full Marks -100 **Time: $2\frac{1}{2}$ Hours**

Answer all questions. All questions carry equal marks.

1. If $\left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right)\left(1-\frac{1}{4^2}\right)\left(1-\frac{1}{5^2}\right)\ \dots\dots\dots.\left(1-\frac{1}{2019^2}\right)=\frac{x}{2019}$, then find the value of x.
यदि $\left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right)\left(1-\frac{1}{4^2}\right)\left(1-\frac{1}{5^2}\right)\ \dots\dots\dots.\left(1-\frac{1}{2019^2}\right)=\frac{x}{2019}$, तो x का मान ज्ञात करें।
Solution: We have $\left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right)\left(1-\frac{1}{4^2}\right)\dots\dots.\left(1-\frac{1}{2019^2}\right)=\frac{x}{2019}$

$$\Rightarrow \frac{1}{2}\cdot\frac{3}{2}\cdot\frac{2}{3}\cdot\frac{4}{3}\cdot\frac{3}{4}\cdot\frac{5}{4}\dots\dots..\cdot\frac{2018}{2019}\cdot\frac{2020}{2019}=\frac{x}{2019}$$

$$\Rightarrow \frac{1}{2}\cdot\frac{2020}{2019}=\frac{x}{2019}$$

$$\Rightarrow x=1010$$

2. If $a^x=(x+y+z)^y,\ a^y=(x+y+z)^z\ and\ a^z=(x+y+z)^x$ then find the value of $x+y+z$, given that $a\neq 0$.
यदि $a^x=(x+y+z)^y,\ a^y=(x+y+z)^z$ तथा $a^z=(x+y+z)^x$ तो $x+y+z$, का मान ज्ञात करें, जबकि $a\neq 0$.

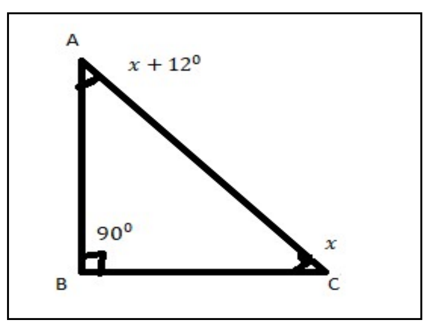
Solution: $a^x.a^y.a^z=(x+y+z)^{x+y+z}$

$$a^{x+y+x}=(x+y+z)^{x+y+z}$$

$$x+y+z=a$$

3. In a right angled triangle, the difference between two acute angle is $(\frac{\pi}{15})^c$. Find the angles in degree.
किसी समकोण त्रिभुज में दोनों न्यून कोणों का अन्तर $(\frac{\pi}{15})^c$ है। कोणों का मान डिग्री में ज्ञात करें।

Solution: $\therefore \angle ACB-\angle BAC=(\frac{\pi}{15})^C$



$$=\frac{\pi}{15}\times\frac{180^0}{\pi}=12^0$$

Let , $\angle ACB=x, \therefore \angle BAC=x+12^0$

$\therefore \boldsymbol{x}+\boldsymbol{x}+12^0=90^0$

$\Rightarrow 2x=78^0$

$\therefore x+12^0=39^0+12^0=51^0$

Hence $\angle DCB=39^0$ and $\angle BAC=51^0$

4. Find natural numbers x, y such that $\sqrt{x} + y = 7$ and $x + \sqrt{y} = 11$.
   दो प्रकृतिक संख्या x, y ज्ञात करें जबकि $\sqrt{x} + y = 7$ तथा $x + \sqrt{y} = 11$.

   Solution:- $\sqrt{x} + y = 7$ ....(1)

   and $x + \sqrt{y} = 11$ ....(2)

   Let $x = a^2$ and $y = b^2$ then from equation (1) is $a + b^2 = 7$ ....(3)
   and equation (2) is $a^2 + b = 11$ ....(4)
   from equation (3)
   $Put\ a = 7 - b^2$
   $(7 - b^2)^2 + b = 11$
   $b^4 - 14b^2 + 49 + b = 11$
   $b^4 - 4b^2 - 10b^2 + 20b - 19b + 38 = 0$
   $b^2(b^2 - 4) - 10b(b - 2) - 19(b - 2) = 0$
   $(b - 2)\{b^3 + 2b^2 - 10b - 19\} = 0$
   $b = 2$
   Put value of $b = 2$ in (3) , we get

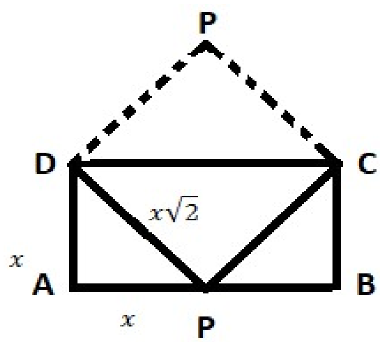
   $a + (2)^2 = 7$
   So $a = 3$
   Therefore $x = a^2 = 9$ and $y = b^2 = 4$

5. If a, b, c are positive numbers such that $abc = 1$, then prove that $a + b + c \geq 3$ and $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} \geq 3$.
   यदि a, b, c धनात्मक संख्यायें हैं जबकि $abc = 1$, तो साबित करें कि $a + b + c \geq 3$ तथा $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} \geq 3$.

   Solution : We know that $\frac{a+b+c}{3} \geq \sqrt[3]{\sqrt{abc}} \geq \frac{3}{\frac{1}{a}+\frac{1}{b}+\frac{1}{c}}$

   $\Rightarrow a + b + c \geq 3$ and $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} \geq 3$

6. The length of a rectangular sheet of paper is twice its breadth. Show how to cut this paper into three pieces which can be rearranged to form a square.
कागज के एक आयताकार पृष्ट की लम्बाई उसकी चौड़ई की दुगुनी है इसे ऐसे तीन टुकड़ों में कैसे काटें कि उन्हें एक वर्ग के आकार में रखा जा सके।



By cutting a ΔAPD and ΔBCP

P is mid-point of AB

7. If $a^2+1=a$, then find the value of $a^{12}+a^6+1$.
यदि $a^2+1=a$ हो तो $a^{12}+a^6+1$ का मान ज्ञात करें।

Given that

$a^2+1=a \quad \Rightarrow a+\frac{1}{a}=1$ ....(1)

Now, $a^{12}+a^6+1=a^6(a^6+1+\frac{1}{a^6})$

$=a^6(a^6+\frac{1}{a^6}+1)$ ....(2)

Now, $= a^6+\frac{1}{a^6} == (a^3)^2+\frac{1}{(a^3)^2}$

$(a^3+\frac{1}{a^3})^2-2=\{(a+\frac{1}{a})^3-3\times 1\times 1\}^2-2$

$\{(1)^3-3\}^2-2=4-2=2$

$\therefore a^6(2+1)=3a^6$

Hence, $a^{12}+a^6+1=3a^6$

We put, $a=1$ it is satisfies the equation

$a^{12}+a^6+1=3$

8. If $x = asec\theta cos\varphi, y = bsec\theta sin\varphi$, $z = ctan\theta$ then find the value of $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2}$.

यदि $x = asec\theta cos\varphi, y = bsec\theta sin\varphi$, $z = ctan\theta$ तो $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2}$ का मान ज्ञात करें।

Solution: Given that

$x = asec\theta. cos\varphi$,$y = bsec\theta. sin\varphi$ and $z = ctan\theta$

$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c} = ?$

$\therefore \frac{x^2}{a^2} = sec^2\theta. cos^2\varphi$, $\frac{y^2}{b^2} = sec^2\theta. sin^2\varphi$ and $\frac{z^2}{c^2} = tan^2\theta$

$\therefore \frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c} = sec^2\theta(cos^2\varphi + sin^2\varphi) - tan^2\theta$

$= sec^2\theta - tan^2\theta = 1$ $[\because (cos^2\varphi + sin^2\varphi) = 1]$

9. Find the digit at the unit place in the number $2019^{2020} + 2020^{2019}$.

संख्या $2019^{2020} + 2020^{2019}$ के इकाई स्थान का अंक ज्ञात करें।

Solution: $9 = 9$

$$9 \times 9 = 81$$

$$9 \times 9 \times 9 = 729$$

$$9 \times 9 \times 9 \times 9 = 6561$$

Hence the value of unit place of $2019^{2020} = 1$

and the value of unit place of $2020^{2019} = 0$

$2019^{2020} + 2020^{2019} \rightarrow$ unit digit $= 1 + 0 = 1$

10. A man has 5 friends. In how many ways can he invite one or more of them in a party?

एक व्यक्ति को 5 मित्र हैं। कितने प्रकार से वह उनमें से एक या अधिक को एक पार्टी में आमंत्रित कर सकता है?

Solution: $5_{C_1} + 5_{C_2} + 5_{C_3} + 5_{C_4} + 5_{C_5}$

$= 5 + \frac{5\times4}{1\times2} + \frac{5\times4}{1\times2} + 5 + 1$

$= 5 + 10 + 10 + 5 + 1$

$= 31$