



BIHAR MATHEMATICAL SOCIETY

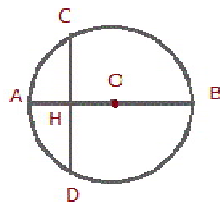
TSTM_SENIOR Model Questions

Full Marks: 100

Full time: 2:30 hours

Write all Questions Answer in your own words, Each question has same marks

- a, b and c are positive whole numbers such that their sum is 117 and
 - a is a prime number
 - b and c are multiple of 'a'
 - $b \leq c$
- If $\sin x + \sin y = a$, and $\cos x + \cos y = b$
Show that $\tan \frac{x}{2}$ and $\tan \frac{y}{2}$ are two roots of the equation $(a^2 + b^2 + 2b)t^2 - 4at + (a^2 + b^2 - 2b) = 0$
- Let a, b, c be three real numbers such that $0 < a, b, c < 1$ and $a + b + c = 2$
Prove that $\frac{a}{1-a} \cdot \frac{b}{1-b} \cdot \frac{c}{1-c} = 8$
- Find the numbers of zeros that appear at the end in the representation of **158!** in base of 10.
Where $n! = n(n-1)(n-2)\dots 1$
- What is the sum of the squares of the roots of the equation?
 $x^2 - 7[x] + 5 = 0$ where $[x]$ represents the maximum integer less than x
- The letters R, M and O represent the whole numbers. If $R \times M \times O = 240$, and $R \times O + M = 46$ what is the value of $R + M + O$?
- If $\frac{1}{\sqrt{2011 + \sqrt{(2011)^2 - 1}}} = \sqrt{m} - \sqrt{n}$,
where m and n are positive integers. What is the value of $m + n$?
- A postman has to deliver five letters to five different houses. Mischievously, he posts one letter through each door without looking to see if it is the correct address. In how many different ways could he do this so that exactly two of the five houses receive the correct letters?
- The length of diameter AB is a two digit integer. Reversing the digits gives the length of a perpendicular chord CD. The distance from their intersection point H to the centre O is a positive rational number. Determine the length of AB.



- In $\triangle PQR$, Show that $\sqrt{\cos \frac{P}{2}} \sqrt{\cos \frac{Q}{2}} \sqrt{\cos \frac{R}{2}} \geq \frac{1}{2\sqrt{3}} (\sqrt{\sin P} + \sqrt{\sin Q} + \sqrt{\sin R})$