Bihar Mathematical Society

Talent Nature Programme (TNP) 2021 (Level III)

Full Marks:- 100 **Time: $2\frac{1}{2}$ Hours**

Answer all questions. All questions carry equal marks.

1. Continuity is necessary but not a sufficient condition for the existence of finite derivative.
2. Evaluate the line integral $\oint_C (y\,dx + z\,dy + x\,dz)$, where C is the curve of intersection of the sphere $x^2 + y^2 + z^2 = a^2$ and the plane $x + z = a$.
3. If a sequence $\{a_n\}$ is convergent, then it converges to a unique limit.
4. Evaluate $\iint_S \vec{F}.\vec{dS}$, where $\vec{F} = 4x\hat{i} - 2y^2\hat{j} + z^2\hat{k}$ and S is the surface bounding the region $x^2 + y^2 = 4, z = 0$ and $z = 3$.
5. Let $\{a_n\}$ and $\{b_n\}$ be two convergent sequences such that $\lim_{n\to\infty} a_n = a$ and $\lim_{n\to\infty} b_n = b$. Then $\lim_{n\to\infty}\left(\frac{a_n}{b_n}\right) = \frac{a}{b}$, provided $b \neq 0$ and $b_n \neq 0$ for any n.
6. Suppose that a function f is analytic inside and on a positively oriented circle C_R, centered at z_0 and with radius R. If M_R denotes maximum value of $|f(z)|$ on C_R, then
$$\left|f^n(z_0)\right| \leq \frac{n!\,M_R}{R^n} \quad (n = 1,2,3,\ldots)$$
7. For the Taylor's polynomial approximation of degree $\leq n$ about the point $x = 0$ for the function $f(x) = e^x$. Determine the value of n such that the error satisfies $|R_n| \leq 0.005$ when $-1 \leq x \leq 1$.
8. Show that the function $f(z) = \sqrt{|xy|}$ is analytic at the origin, although the Cauchy – Riemann equations are satisfied at that point.
9. Let R be a ring with unit element $e \in R$. Then the ring R, when considering as a right R-module is isomorphic to ring $Hom_R(M, M)$.
10. Hom_F (V, U) $\simeq F^{m\times n}$ as a vector space over F.